

1. Let $A = (-\infty, 0) \cup (0, \infty)$. Define $f: A \rightarrow \mathbb{R}$ by $f(x) = \sin\left(\frac{1}{x}\right)$

(a) Sketch a graph of f

(b) Use Corollary 4.2.5 to show $\lim_{x \rightarrow 0} f(x)$ does not exist

Hint: Consider $(x_n) = \left(\frac{1}{2\pi n}\right)$ and $(y_n) = \left(\frac{1}{2\pi n + \frac{\pi}{2}}\right)$

2. Prove that $f(x) = 2x + 3$ is continuous at $x = 4$

3. Prove that $g(x) = x^2$ is continuous at $x = 4$

Theorem 4.3.4 (Algebraic Continuity Theorem)

Assume $f : A \rightarrow \mathbb{R}$ and $g : A \rightarrow \mathbb{R}$ are continuous at $c \in A$. Then

- (i) $kf(x)$ is continuous for all $k \in \mathbb{R}$
- (ii) $f(x) + g(x)$ is continuous at c
- (iii) $f(x)g(x)$ is continuous at c
- (iv) $f(x)/g(x)$ is continuous at c if the quotient is defined