Problem Set #7

Due Tuesday, December 7, 2021 @ midnight
Submit as single pdf file to onCourse

Remember to review the Guidelines for Problem Sets on the course webpage. This is an individual problem set, but you may discuss this with other students in the class as long as you cite them on your Problem Set.

1. Let $A = \begin{bmatrix} 42 & -3 & 24 & -6 \\ -30 & 33 & 168 & -42 \\ 236 & -4 & 131 & -53 \\ 1064 & -40 & 500 & -206 \end{bmatrix}$

   (a) Find the eigenvalues of A. Using Mathematica is fine.

   (b) Use your answer from (a) to explain how you know A is diagonalizable.

   (c) Diagonalize A. That is, write $A = PDP^{-1}$ where D is diagonal.

2. Let W be the subspace of $\mathbb{R}^3$ spanned by $\vec{\mathbf{u}}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$ and $\vec{\mathbf{u}}_2 = \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}$ and let $\vec{\mathbf{y}} = \begin{bmatrix} 3 \\ 3 \\ -4 \end{bmatrix}$.

   (a) Verify that $\{\vec{\mathbf{u}}_1, \vec{\mathbf{u}}_2\}$ is an orthogonal basis for W.

   (b) Use the Orthogonal Decomposition Theorem to write $\vec{\mathbf{y}} = \hat{y} + \vec{\mathbf{z}}$ where $\hat{y} \in W$ and $\vec{\mathbf{z}} \in W^{\perp}$.

3. Let $A = \begin{bmatrix} 4 & -2 & -6 & 4 \\ -4 & 8 & -1 & 0 \\ -7 & -2 & 10 & -8 \\ 3 & -6 & 0 & -4 \\ 3 & 4 & -8 & 7 \\ 10 & -7 & -2 & 10 \\ 6 & 10 & -2 & -1 \end{bmatrix}$ and $\vec{\mathbf{b}} = \begin{bmatrix} 4 \\ -1 \\ 1 \\ 10 \\ -4 \\ -4 \\ -6 \end{bmatrix}$

   (a) Show that the system $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ is inconsistent.

   (b) Find the least squares solution $A\hat{x} = \hat{b}$.

4. Let $A = \begin{bmatrix} \frac{733}{325} & -\frac{2}{13} & -\frac{306}{325} \\ -\frac{2}{13} & \frac{53}{26} & \frac{3}{26} \\ -\frac{306}{325} & \frac{3}{26} & \frac{1109}{650} \end{bmatrix}$

   (a) Explain how you know that A is orthogonally diagonalizable.

   (b) Orthogonally diagonalize A. That is, write $A = PDP^T$ where D is diagonal.