Problem Set #5

Due Thursday, October 28, 2021 @ midnight
Submit as single pdf file to onCourse

Remember to review the Guidelines for Problem Sets on the course webpage.

1. Let $A = \begin{bmatrix} -6 & 24 & 47 \\ 12 & 8 & -45 \\ -12 & -24 & 31 \end{bmatrix}$

   (a) Give a basis for $\text{nul}(A)$ and describe $\text{nul}(A)$ geometrically.

   (b) Give a basis for $\text{col}(A)$ and describe $\text{col}(A)$ geometrically.

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$ where $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$.

   (a) Find a basis for $\ker(T)$ and describe $\ker(T)$ geometrically.

   (b) Find a basis for $\text{range}(T)$ and describe $\text{range}(T)$ geometrically.

   (c) Describe T in geometric terms.
   e.g. "T rotates the plane by $\frac{\pi}{3}$ radians counter-clockwise", or "T projects the plane onto the x-axis", etc.
   To be clear, T doesn't do either of these, but these are examples of how you can describe T.
   It may be useful to pick a few specific points in $\mathbb{R}^2$ and see what their image is under T.

3. Show that $\mathcal{B} = \left\{4 - 31t - 7t^2,\ 1 + t,\ 5t + t^2\right\}$ is *not* a basis for $\mathbb{P}_2$.

4. (a) Let $S = \{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_k\}$ be a set of vectors in $\mathbb{R}^n$ with $k < n$. Use a theorem from earlier in the semester to explain why S cannot be a basis for $\mathbb{R}^n$.

   (b) Let $S = \{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_k\}$ be a set of vectors in $\mathbb{R}^n$ with $k > n$. Use a theorem from earlier in the semester to explain why S cannot be a basis for $\mathbb{R}^n$.

   (These problems are essentially the same as Exercises 4.3.29 and 4.3.30 from the text, Lay's Linear Algebra, 4th edition)