Problem Set #1

Due Thursday, September 9, 2021 @ midnight
Submit as single pdf file to onCourse

Remember to review the Guidelines for Problem Sets on the course webpage.

1. Find the general solution of the system whose augmented matrix is

$$\left[\begin{array}{cccc|c} -1 & -3 & -10 & -1 & -14 \\ -2 & 1 & 1 & 0 & 1 \\ 2 & 4 & 14 & 1 & 21 \end{array}\right]$$

   For this problem, perform the row reduction by hand and document each step in the process.

2. Find an equation involving g, h, and k that makes this augmented matrix correspond to a consistent system:

$$\left[\begin{array}{ccc|c} 1 & 3 & -5 & g \\ 0 & 2 & -1 & h \\ -2 & 4 & 5 & k \end{array}\right]$$

   (The problem is very similar to Exercise 1.1.25 from the text, Lay's Linear Algebra, 4th edition)

3. Suppose a 4×6 *coefficient* matrix for a system has four pivot columns. Is the system consistent? Why or why not?

   (The problem is essentially the same as Exercise 1.1.26 from the text, Lay's Linear Algebra, 4th edition)

4. In a wind tunnel experiment, the force on a projectile due to air resistance was measured at different velocities:

Velocity (100 ft/sec)	0	2	4	6	8	10
Force (100 lb)	0	2.90	14.8	39.6	74.3	119

   (a) Set up the system of six equations in six unknowns to find the interpolating polynomial of degree five for these data, $p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$.

   (b) What is the polynomial $p(t)$? You can use technology to perform the row reduction.

   (c) Graph the points and your polynomial on the same axes to verify that the polynomial goes through all of the points.

   (d) Use your polynomial to estimate the force when the projectile is moving at 750 ft/sec.

   (e) What happens if you try to use a cubic polynomial rather than a polynomial of degree 5?

   (The problem is essentially the same as Exercise 1.2.34 from the text, Lay's Linear Algebra, 4th edition. It may be useful to read the description before exercise 1.2.33.)

5. Determine if $\vec{\mathbf{b}}$ is a linear combination of the vectors formed from the columns of the matrix A.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 1 & 4 & 5 \end{bmatrix}, \qquad \vec{\mathbf{b}} = \begin{bmatrix} 16 \\ -3 \\ 21 \end{bmatrix}$$

   You can use technology for the computations.

   (The problem is very similar to Exercise 1.3.14 from the text, Lay's Linear Algebra, 4th edition)