The Invertible Matrix Theorem (Theorem 2.8)
Let A be a square $n \times n$ matrix. Then the following statements are equivalent. That is, for a given A, the statements are either all true or all false.

a. A is an invertible matrix.

b. A is row equivalent to the $n \times n$ identity matrix.

c. A has n pivot positions.

d. The equation $A\vec{\mathbf{x}} = \vec{\mathbf{0}}$ has only the trivial solution.

e. The columns of A form a linearly independent set.

f. The linear transformation $\vec{\mathbf{x}} \longrightarrow A\vec{\mathbf{x}}$ is one-to-one.

g. The equation $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$. has at least one solution for each $\vec{\mathbf{b}}$ in $\mathbb{R}^n$.

h. The columns of A span $\mathbb{R}^n$.

i. The linear transformation $\vec{\mathbf{x}} \longrightarrow A\vec{\mathbf{x}}$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.

j. There is an $n \times n$ matrix C such that $CA = I$.

k. There is an $n \times n$ matrix D such that $AD = I$.

l. A^T is an invertible matrix.

Recall Theorems 1.4 and 1.12:

Theorem 1.4
Let A be an $m \times n$ matrix. The following are equivalent:

a. For all $\vec{\mathbf{b}} \in \mathbb{R}^m$, $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ has a solution.

b. Each $\vec{\mathbf{b}} \in \mathbb{R}^m$ is a linear combination of the columns of A.

c. The columns of A span $\mathbb{R}^m$

d. A has a pivot in every row.

Theorem 1.12
Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear transformation and A the standard matrix for T.

a. T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ iff the columns of A span $\mathbb{R}^m$

b. T is one-one iff the columns of A are linearly independent

Sketch of proof of the Invertible Matrix Theorem