From Lay, Section 1.9

THEOREM 12

Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation and let A be the standard matrix for T. Then:

a. T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ if and only if the columns of A span $\mathbb{R}^m$;

b. T is one-to-one if and only if the columns of A are linearly independent.

THEOREM 2

Let A be an $m \times n$ matrix, and let B and C have sizes for which the indicated sums and products are defined.

a. $A(BC) = (AB)C$ (associative law of multiplication)

b. $A(B + C) = AB + AC$ (left distributive law)

c. $(B + C)A = BA + CA$ (right distributive law)

d. $r(AB) = (rA)B = A(rB)$ for any scalar r

e. $I_m A = A = A I_n$ (identity for matrix multiplication)

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 3 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -4 \\ 3 & -6 \end{bmatrix}$, **and** $D = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & 3 \end{bmatrix}$

1. Compute AB and BA
2. Compute AC and BC
3. Compute AD and DA
4. What interesting properties of matrix multiplication do these examples demonstrate?

Let $F = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 2 & 3 \\ 0 & 4 & -7 \end{bmatrix}$, $G = \begin{bmatrix} 3 & -2 & 4 \\ 7 & 12 & 8 \\ 16 & 2 & 3 \end{bmatrix}$, $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Feel free to use today's Mathematica notebook for these computations.

5. Find F^{-1} and G^{-1}
6. Compare the following products: $F^{-1}G^{-1}$, $G^{-1}F^{-1}$, $(FG)^{-1}$, $(GF)^{-1}$
7. Compare $(FG)^T$, F^TG^T, and G^TF^T
8. Find $\left(F^T\right)^{-1}$
9. Compare F, E_1F, and E_2E_1F

What general observations can you make from your computations?