1. Solve the following discrete log problems:

   (a) $11^x \equiv 9 \mod 31$

   (b) $3^x \equiv 24 \mod 31$

   (c) $2^x \equiv 27 \mod 31$

2. Let $p = 11$

   (a) What are the possible orders for elements in $\mathbb{Z}_p^*$?

   (b) Find a generator α of $\mathbb{Z}_p^*$.

   (c) Fill in the following table:

k	$\alpha^k \mod p$	$\text{ord}(\alpha^k)$
1		
2		
$\vdots$		
10		

   (d) For which values of k is α^k a generator?

   (e) How are the values in your last answer related to $\phi(p)$?

   (f) How many generators does $\mathbb{Z}_p^*$ have?

3. Repeat the previous problem with $p = 23$. Note that your table will have 22 rows.

   The Mathematica command MultiplicativeOrder[ ] might be handy.

4. Show that $p = 1\,786\,511$ is a poor choice as the modulus for Diffie-Hellman Key Exchange.

   The Mathematica commands PrimeQ[ ] and FactorInteger[ ] may be useful.

5. Show that $p = 1\,786\,553$ is a reasonable choice for DHKE and find an appropriate value α.