Addition on elliptic curve $E : Y^2 = X^3 + AX + B$

Let P_1 and P_2 be two points on E

- If $P_1 = \mathcal{O}$, then $P_1 + P_2 = P_2$
  If $P_2 = \mathcal{O}$, then $P_1 + P_2 = P_1$
- Otherwise, write $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$
  - If $x_1 = x_2$ and $y_1 = -y_2$, then $P_1 = -P_2$ in E and $P_1 + P_2 = \mathcal{O}$
  - Otherwise, define

$$\lambda = \begin{cases} \dfrac{y_2 - y_1}{x_2 - x_1} & \text{if } P_1 \neq P_2 \\ \dfrac{3x_1^2 + A}{2y_1} & \text{if } P_1 = P_2 \end{cases}$$

and let

$$x_3 = \lambda^2 - x_1 - x_2 \quad \text{and} \quad y_3 = \lambda(x_1 - x_3) - y_1$$

Then $P_1 + P_2 = (x_3, y_3)$