Pollard's ρ applied to DLP $g^x \equiv h \mod p$

1. Define $f : \mathbb{F}_p \to \mathbb{F}_p$: $$f(x) = \begin{cases} gx & \text{if } 0 \le x < p/3 \\ x^2 & \text{if } p/3 \le x < 2p/3 \\ hx & \text{if } 2p/3 \le x < p \end{cases}$$

2. Define sequence $x_0 = 1$, $x_{i+1} = f(x_i) = g^{\alpha_i} h^{\beta_i}$ where
$$\alpha_{i+1} = \begin{cases} \alpha_i + 1 & \text{if } 0 \le x_i < p/3 \\ 2\alpha_i & \text{if } p/3 \le x_i < 2p/3 \\ \alpha_i & \text{if } 2p/3 \le x_i < p \end{cases} \qquad \beta_{i+1} = \begin{cases} \beta_i & \text{if } 0 \le x_i < p/3 \\ 2\beta_i & \text{if } p/3 \le x_i < 2p/3 \\ \beta_i + 1 & \text{if } 2p/3 \le x_i < p \end{cases}$$

3. Look for collision in sequences $\{x_i\} = \left\{g^{\alpha_i} h^{\beta_i}\right\}$ and $\{y_i\} = \{x_{2i}\} = \left\{g^{\gamma_i} h^{\delta_i}\right\}$

4. This gives $g^u \equiv h^v \mod p$. Take v-th root