These are only a *few* sample problems to *help* you prepare for the exam. You should also be certain that you completely understand the WeBWorK assignments, Problems Sets, in-class work, and your class notes.

1. Show that the improper integral $\displaystyle\int_1^\infty e^{-x^3} x^5 \, dx$ converges and find its exact value.

2. Explain why the integral is improper and determine whether the integral converges or diverges. You do not need to find the values of the convergent integrals.

$$\text{(a)} \int_2^\infty \frac{x}{x^2-2}\, dx \qquad \text{(b)} \int_0^{1/2} \frac{1}{\sqrt[3]{x}-x}\, dx$$

3. Do the following series diverge, converge conditionally, or converge absolutely? Be sure to justify your answer by giving reference to the appropriate theorems and/or tests.

$$(a) \sum_{k=3}^{\infty} (-1)^{k+1} \frac{\sqrt{k}}{\ln(k)} \qquad (c) \sum_{j=0}^{\infty} \cos(j)\, j^5\, e^{-j^3}$$

$$(b) \sum_{k=3}^{\infty} \frac{2k}{4k^8+7k^2+6} \qquad (d) \sum_{k=10}^{\infty} (-1)^k \frac{3}{4k+2}$$

Pick one of the series that converges. How close does S_{75} approximate the value of the series?

4. Let $\mathcal{I} = \displaystyle\int_0^1 \cos(x^2)\, dx$.
Calculate $\mathcal{I}$ accurate within 0.001 by hand with the use of only a four function $(+, -, \times, \div)$ calculator.

5. Find the *exact* value of the following by hand (no decimal approximations!)

$$(a) \sum_{k=42}^{\infty} \frac{1}{4^k} \qquad (c) \sum_{k=0}^{\infty} \frac{1}{3^k\, k!}$$

$$(b) \sum_{k=0}^{326} \left(\frac{3}{7}\right)^k \qquad (d) \sum_{k=0}^{\infty} (-1)^k \frac{(\pi/3)^{2k}}{(2k)!}$$

6. Let $I = \displaystyle\int_a^b f(x)\, dx$ be an improper integral where b may be a finite number or infinity.

   (a) Give values of a and b, and a function $f(x)$ where
   - The integrand $f(x)$ converges on the interval of integration
   - The integral I diverges

   (b) Give values of a and b, and a function $f(x)$ where
   - The integrand $f(x)$ diverges on the interval of integration
   - The integral I converges

   (c) Be sure to explain your answers

7. (a) Is it possible that the terms of a series converge but the partial sums diverge? Explain.
   (b) Can the partial sums converge but the terms diverge? Explain.