Determine whether each of the following improper integrals converges or diverges. If the integral converges, find its exact value.

1. $\displaystyle\int_1^\infty \frac{1}{1+x^2}\,dx$

2. $\displaystyle\int_1^\infty \frac{1}{x\ln(x)}\,dx$

3. $\displaystyle\int_0^\infty x\,e^{-x^2}\,dx$

4. $\displaystyle\int_0^\infty x\,e^{-x}\,dx$

5. $\displaystyle\int_0^{\pi/2} \tan(x)\,dx$