

These are only a *few* sample problems to *help* you prepare for the exam. You should also be certain that you completely understand the WeBWorK assignments, Problems Sets, in-class work, and your class notes.

1. The contour plot of $z = f(x, y)$ is shown below.

(a) The following points are critical points of f :

$$p_1 = (0, 0)$$

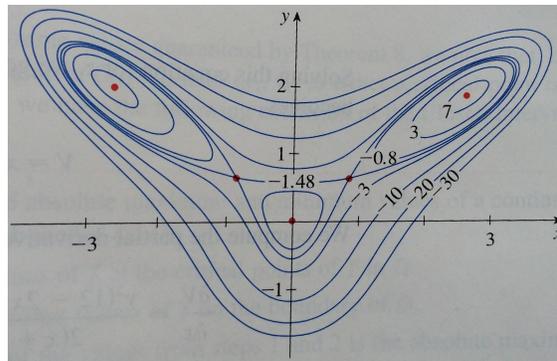
$$p_2 = (-2.644, 1.898)$$

$$p_3 = (2.644, 1.898)$$

$$p_4 = (-0.856, 0.646)$$

$$p_5 = (0.856, 0.646)$$

Use the contour plot to classify the critical points.



(b) If you are standing on the surface at the point $(2, 2)$ and move in the direction $\langle 1, -1 \rangle$, will the directional derivative be positive, negative or zero?

(c) What if you are standing at the point $(0, -1)$ and move in the $\langle -1, 0 \rangle$ direction?

(d) What if you are standing at the point $(0, -1)$ and move in the $\langle 0, 1 \rangle$ direction?

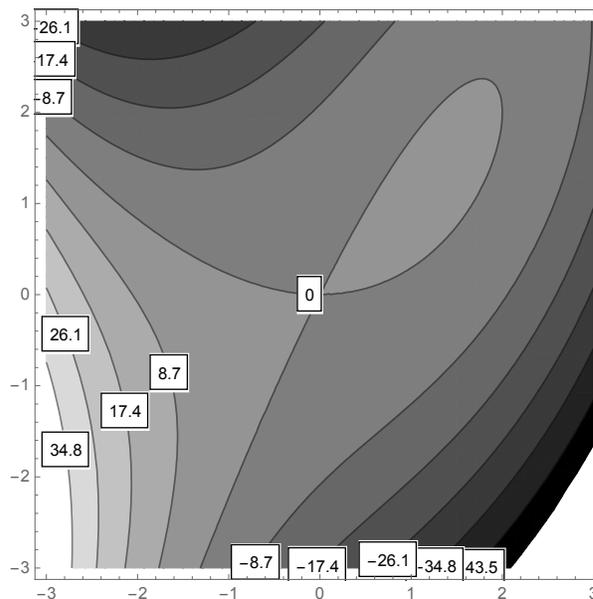
2. Let $f(x, y) = 4xy - x^3 - 2y^2$

(a) Find and classify the critical points of $f(x, y)$

(b) What is the rate of change of $f(x, y)$ at the point $(1, 2)$ in the direction of $\vec{v} = \langle 1, 3 \rangle$?

(c) If you are standing at the point on the surface with $(x, y) = (1, 2)$ and drop your glass of milk, in what direction will your spilt milk flow?

(d) Suppose you are standing at the point on the surface with $(x, y) = (1, 2)$. Give a direction you could walk to maintain the same altitude (i.e. height above the xy -plane). Is there another direction you could have walked?



3. Evaluate the following integrals.

(a) $\int_0^8 \int_{\sqrt[3]{y}}^2 \sin(x^4) dx dy$

(b) $\int_0^1 \int_{\sqrt[3]{y}}^1 \cos(x^3) dx dy$

4. Let $f(x, y) = y \cos(x^2) + 3$ and let R be the region in the xy -plane bounded by the graphs $x = y^2$ and $x = 9$. Find the volume of the solid that lies below the graph $z = f(x, y)$ and above R .

5. Find the point on the surface $z^2 = 10 - 2x + x^2 - 6y + y^2$ that is closest to the origin.

6. If $\nabla f = \langle -2x \sin(x^2) + 8xy + 2y^2, 4x^2 + 4xy + 2y + \cos(y) \rangle$, find $f(x, y)$.