

1. For each transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, find the corresponding matrix A .

(a) T rotates by $\frac{\pi}{3}$ counter-clockwise and then reflects over the line $y = x$

(b) T reflects over the line $y = x$ and then rotates by $\frac{\pi}{3}$ counter-clockwise

(c) What is the image of $\mathbf{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ under each transformation?

2. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 3 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & -4 \\ 3 & -6 \end{bmatrix}$

(a) Compute AC and BC

(b) What interesting property of matrix multiplication does this example demonstrate?

$$\text{Let } A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 2 & 3 \\ 0 & 4 & -7 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 & 4 \\ 7 & 12 & 8 \\ 16 & 2 & 3 \end{bmatrix}, E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Feel free to use *Mathematica* for these problems.

1. Find A^{-1} and B^{-1}

2. Compare the following products:

$$A^{-1}B^{-1}, \quad B^{-1}A^{-1}, \quad (AB)^{-1}, \quad (BA)^{-1}$$

3. Compare $(AB)^T$, $A^T B^T$, and $B^T A^T$

4. Find $(A^T)^{-1}$

5. Compare A , $E_1 A$, and $E_2 E_1 A$