1. Let $A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & -6 & 7 \end{bmatrix}$.

   (a) Find all solutions to the homogeneous system $A\mathbf{x} = \mathbf{0}$.

   (b) Find all solutions to $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$.

2. Find all solutions to $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \\ 4 & 8 & 7 & 11 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -9 \\ -13 \\ -31 \end{bmatrix}$$

3. Create an example of a matrix A and vector $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions and $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

1. Are the columns of $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -3 & 4 \\ -1 & 3 & 2 \end{bmatrix}$ linearly independent or linearly dependent?

2. Do the vectors $\mathbf{v_1} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$ $\mathbf{v_2} = \begin{bmatrix} 0 \\ -1 \\ 6 \end{bmatrix}$ $\mathbf{v_3} = \begin{bmatrix} -2 \\ -4 \\ 21 \end{bmatrix}$ lie in the same plane in $\mathbb{R}^3$?

3. If A is a 4×5 matrix, are the columns linearly independent or linearly dependent? What if A is 5×4?

4. If the system $A\mathbf{x} = \mathbf{b}$ has a unique solution, are the columns of A linearly independent or linearly dependent? Why?