Answer True / False

Let
$$\begin{aligned}
\mathbf{v_1} &= (1, 3, 18, 2) \\
\mathbf{v_2} &= (2, -1, 9, 0) \\
\mathbf{v_3} &= (3, 2, -4, 1) \\
\mathbf{v_4} &= (4, 7, 1, 3)
\end{aligned}$$
and
$$A = \begin{bmatrix} 1 & 3 & 2 \\ -2 & 1 & 4 \\ 6 & 2 & 1 \\ 5 & -17 & 32 \end{bmatrix}$$

1. The vectors $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ span $\mathbb{R}^4$

2. The columns of A span $\mathbb{R}^4$

3. The equation $B\mathbf{x} = \mathbf{b}$ has a unique solution where $B = [\mathbf{v_1}\ \mathbf{v_2}\ \mathbf{v_3}\ \mathbf{v_4}]$ and $\mathbf{b} = (72, -128, \pi, e^{-411})$

4. There exists $\mathbf{b} \in \mathbb{R}^4$ such that $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

1. Let $A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & -6 & 7 \end{bmatrix}$.

   (a) Find all solutions to the homogeneous system $A\mathbf{x} = \mathbf{0}$.

   (b) Find all solutions to $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$.

2. Find all solutions to $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \\ 4 & 8 & 7 & 11 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -9 \\ -13 \\ -31 \end{bmatrix}$$

3. Create an example of a matrix A and vector $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions and $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.