$$\left[\begin{array}{ccc|c} 2 & 4 & -6 & 6 \\ 4 & 8 & -9 & 14 \\ 2 & 4 & -3 & 8 \end{array}\right] \xrightarrow{R2-2R1} \left[\begin{array}{ccc|c} 2 & 4 & -6 & 6 \\ 0 & 0 & 3 & 2 \\ 2 & 4 & -3 & 8 \end{array}\right] \xrightarrow{R3-R1} \left[\begin{array}{ccc|c} 2 & 4 & -6 & 6 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 3 & 2 \end{array}\right]$$

$$\xrightarrow{R3-R2} \left[\begin{array}{ccc|c} 2 & 4 & -6 & 6 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 4 & -6 & 6 \\ 4 & 8 & -9 & 14 \\ 2 & 4 & -3 & 8 \end{array}\right] \xrightarrow{R2-2R1} \left[\begin{array}{ccc|c} 2 & 4 & -6 & 6 \\ 0 & 0 & 3 & 2 \\ 2 & 4 & -3 & 8 \end{array}\right] \xrightarrow{R3-R1} \left[\begin{array}{ccc|c} 2 & 4 & -6 & 6 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 3 & 2 \end{array}\right]$$

$$\xrightarrow{R3-R2} \left[\begin{array}{ccc|c} 2 & 4 & -6 & 6 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Echelon Form
but not
Reduced Echelon Form

$$\left[\begin{array}{ccc|c} 2 & 4 & -6 & 6 \\ 4 & 8 & -9 & 14 \\ 2 & 4 & -3 & 8 \end{array}\right] \xrightarrow{R2-2R1} \left[\begin{array}{ccc|c} 2 & 4 & -6 & 6 \\ 0 & 0 & 3 & 2 \\ 2 & 4 & -3 & 8 \end{array}\right] \xrightarrow{R3-R1} \left[\begin{array}{ccc|c} 2 & 4 & -6 & 6 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 3 & 2 \end{array}\right]$$

$$\xrightarrow{R3-R2} \left[\begin{array}{ccc|c} 2 & 4 & -6 & 6 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R1+2R2} \left[\begin{array}{ccc|c} 2 & 4 & 0 & 10 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Pivots

$$\xrightarrow{\frac{1}{2}R1 \quad \text{and} \quad \frac{1}{3}R2} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Find the general solutions of the system whose augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 3 & 2 & 5 \end{array}\right]$$

Let $\mathbf{u} = (2, 1, 1)$ and $\mathbf{v} = (5, 1, -2)$

1. (a) Does $\mathbf{b} = (-14, -1, 11)$ lie in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$?

   (b) What does this tell you about the lines

$$2x + 5y = -14, \quad x + y = -1, \quad \text{and} \quad x - 2y = 11?$$

2. (a) Does $\mathbf{b} = (13, 8, -42)$ lie in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$?

   (b) What does this tell you about the lines

$$2x + 5y = 13, \quad x + y = 8, \quad \text{and} \quad x - 2y = -42?$$