1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that rotates the plane by $\frac{\pi}{3}$ counter-clockwise and projects onto the $y$-axis. Define

$$\ker(T) = \{\mathbf{x} \in \mathbb{R}^2 \mid T(\mathbf{x}) = \mathbf{0}\}$$
$$\text{range}(T) = \{\mathbf{b} \in \mathbb{R}^2 \mid \exists\ \mathbf{x} \in \mathbb{R}^2 \ni\ T(\mathbf{x}) = \mathbf{b}\}$$

   (a) Find a basis for $\ker(T)$

   (b) Find a basis for $\text{range}(T)$

2. Let $\mathbf{u} = (1, 0, 4)$, $\mathbf{v} = (2, -1, 1)$, $\mathbf{w} = (-1, 2, 7)$

   (a) Show that the set $\mathcal{B} = \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ forms a basis for $\mathbb{R}^3$.

   (b) Write $\mathbf{b} = (17, -2, 4)$ as a linear combination of the vectors in $\mathcal{B}$

3. Let $p_1(t) = 1 + t^2$, $p_2(t) = 2 - t + 3t^2$, and $p_3(t) = -1 + 2t - t^2$

   (a) Show that the set $\mathcal{B} = \{p_1, p_2, p_3\}$ forms a basis for $\mathbb{P}_2$

   (b) Write $p(t) = 3 + 6t - 7t^2$ as a linear combination of the vectors in $\mathcal{B}$