A **vector space** is a nonempty set of objects V, called *vectors*, which have two operations defined: *addition* of vectors and *multiplication by scalars* (real numbers), subject to the ten axioms listed below. The axioms must hold for all vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w} \in V$ and for all scalars c and d.

1. $\mathbf{u} + \mathbf{v} \in V$
2. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
3. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
4. There exists a vector $\mathbf{0} \in V$ such that $\mathbf{u} + \mathbf{0} = \mathbf{u}$
5. For all $\mathbf{u} \in V$, there is a vector $-\mathbf{u} \in V$ such that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
6. $c\mathbf{u} \in V$
7. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
8. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
9. $c(d\mathbf{u}) = (cd)\mathbf{u}$
10. $1\mathbf{u} = \mathbf{u}$

1. Let $H_1 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; x \geq 0, y \geq 0 \right\}$ be the first quadrant in $\mathbb{R}^2$.

   Show that H_1 is *not* a subspace of $\mathbb{R}^2$ by finding a specific vector $\mathbf{u} \in H_1$ and a specific scalar c such that $c\mathbf{u} \notin H_1$.

2. Let $H_2 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; x \cdot y \geq 0 \right\}$ be the union of the first and third quadrants in $\mathbb{R}^2$.

   Show that H_2 is *not* a subspace of $\mathbb{R}^2$ by finding specific vectors $\mathbf{u}, \mathbf{v} \in H_2$ such that $\mathbf{u} + \mathbf{v} \notin H_2$.