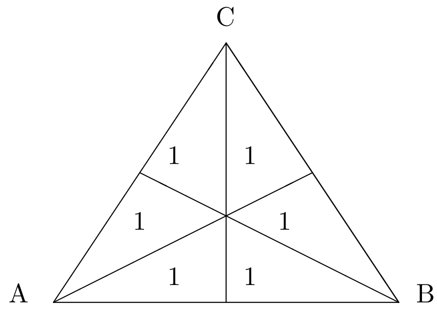
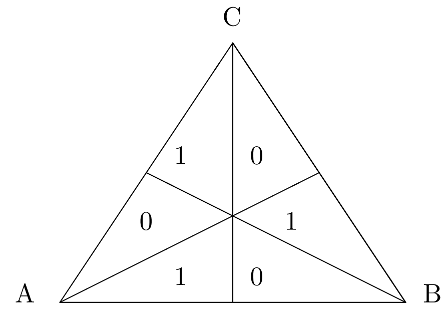
The Decomposition of $\mathbb{R}^6$ for Three Candidate Elections

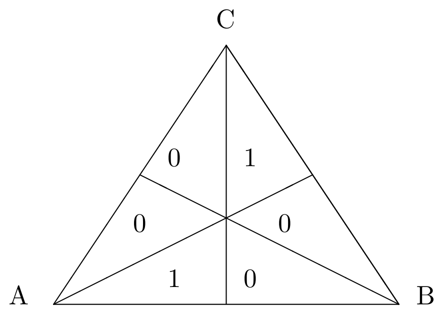
$K=$



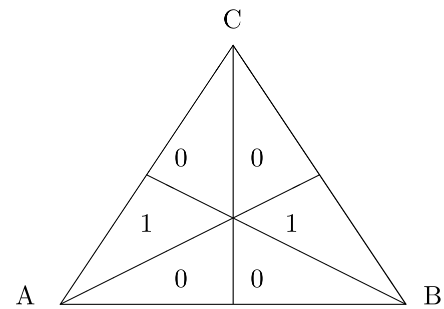
$C_y=$



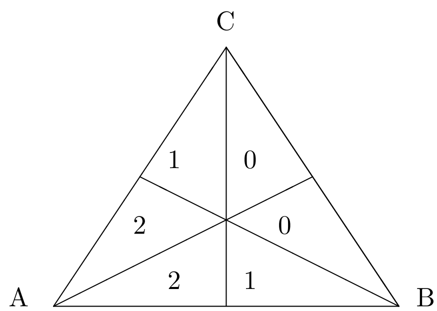
$R_1=$



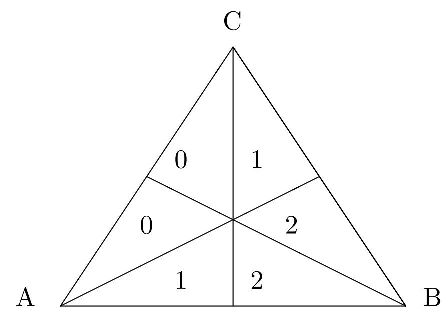
$R_2=$



$B_1=$



$B_2=$



$$M = \begin{bmatrix} 1 & 1 & 1 & 0 & 2 & 1 \\ 1 & 0 & 0 & 1 & 2 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} \qquad M^{-1} = \frac{1}{6}\begin{bmatrix} -4 & 1 & 5 & 4 & -1 & 1 \\ 2 & -2 & 2 & -2 & 2 & -2 \\ 3 & 0 & -3 & 3 & 0 & -3 \\ 0 & 3 & -3 & 0 & 3 & -3 \\ 2 & 1 & -1 & -2 & -1 & 1 \\ 1 & -1 & -2 & -1 & 1 & 2 \end{bmatrix}$$

- Columns of M are formed by $K, C_y, B_A, B_B, R_A, R_B$
- $\det(M) = 12 \neq 0$ implies that $K, C_y, B_A, B_B, R_A, R_B$ form a basis for $\mathbb{R}^6$
- If $\vec{\mathbf{p}} \in \mathbb{R}^6$ is a profile with standard coordinates, then
  $M^{-1}\vec{\mathbf{p}}$ gives the decomposition in terms of $K, C_y, R_1, R_2, B_1, B_2$