Homework #5

Due Friday, April 11, 2014 at 11:30 am

1. For a given profile, let q_0 denote the plurality point and q_1 the antiplurality point in the representation simplex.

   (a) In each case, create a profile with these outcomes or explain why it is impossible.
   If the profile exists, briefly explain your thought process for creating the example.

   i. $q_0 = (\frac{1}{4}, \frac{5}{12}, \frac{1}{3})$ $q_1 = (\frac{3}{8}, \frac{1}{4}, \frac{3}{8})$

   ii. $q_0 = (\frac{1}{32}, \frac{2}{3}, \frac{29}{96})$ $q_1 = (\frac{1}{5}, \frac{8}{15}, \frac{4}{15})$

   (b) For each profile that you determined was possible in (a), give all possible outcomes for this profile (including ties) using a weighted method.

2. Create examples of profiles with three candidates that have the following properties, or explain why no such profile exists. If the profile exists, for each outcome, give the weights for a method that determines that outcome.

   (a) All weighted methods give the ranking of $A > B > C$.

   (b) By varying the weighted method, the profile gives **only** the following *strict* outcomes (other outcomes involving ties are allowed) :

   $A > B > C, \quad A > C > B, \quad C > A > B$

   (c) Repeat part (b) but with outcomes

   $C > A > B, \quad C > B > A, \quad A > B > C$

   (d) Repeat part (b) but with outcomes

   $C > B > A, \quad A > B > C$

3. Consider the profile

10	$A > B > C$	7	$C > B > A$
7	$A > C > B$	9	$B > C > A$
7	$C > A > B$	7	$B > A > C$

   (a) Find the decomposition of this profile in terms of $K, C_y, R_1, R_2, B_1, B_2$.

   (b) Determine the point in pairwise space corresponding to the profile.

   (c) Determine the point on the transitivity plane corresponding to the profile.

   (d) Explain how your results show that the Borda Count and pairwise outcomes for this profile differ.

4. Create examples of profiles with three candidates that have the following properties, or explain why no such profile exists.

   Explain how you constructed the profile – Guess and check is *not* ok.

   (a) The Borda count gives $C > B > A$, the pairwise outcome is a cycle $A > B > C > A$, and plurality and antiplurality give the opposite outcomes of $C > A > B$ and $B > A > C$, respectively.

   (b) The Borda count gives $A > B > C$ and C is the Condorcet winner.

   (c) The pairwise outcome is $A > C > B$, the Borda count gives $A > B > C$, plurality gives $C > A > B$, and using weights $(5, 2, 0)$ gives $B > C > A$.