

Why the product rule is true:

$$\begin{aligned} & \frac{d}{dx} (f(x) g(x)) \\ &= \lim_{h \rightarrow 0} \frac{f(x+h) g(x+h) - f(x) g(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(x+h) g(x+h) - \color{red}{f(x) g(x+h)} + \color{red}{f(x) g(x+h)} - f(x) g(x)}{h} \\ &= \lim_{h \rightarrow 0} \left(\frac{f(x+h) \color{blue}{g(x+h)} - f(x) \color{blue}{g(x+h)}}{h} + \frac{\color{blue}{f(x) g(x+h)} - \color{blue}{f(x) g(x)}}{h} \right) \\ &= \lim_{h \rightarrow 0} \left(\left[\frac{f(x+h) - f(x)}{h} \right] \color{blue}{g(x+h)} + f(x) \left[\frac{\color{blue}{g(x+h)} - \color{blue}{g(x)}}{h} \right] \right) \\ &= f'(x) \color{blue}{g(x)} + \color{blue}{f(x)} g'(x) \end{aligned}$$

1. For each function, find its derivative.

$$(a) \quad h(x) = (x^{3/2} - 4x) \left(\sqrt{x} + \frac{2}{x^3} \right)$$

$$(b) \quad h(x) = \frac{3 + 2x^{-3}}{8x^3 - 4x}$$

$$(c) \quad h(x) = \frac{(x^2 - 2x) \left(x^{10} - \frac{1}{x} \right)}{x^2}$$

2. Using the function from 1(b), find the equation of the line tangent to $y = h(x)$ at $x = 1$.