While Visions of Roadrunners Danced in His Head

Wile E. Coyote
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Dear Calculus Students:

HELP ME!! For the last two months, I've had this series of recurring nightmares that are about to drive me out of my mind! When I went looking for help, your enterprising and resourceful professor naturally referred me to you.

The scenario is nearly always the same. I'm standing at the end of a road that is 1 kilometer long (for some reason the road has those little green kilometer markers on it), and there at the other end is that `@!*^@#!` Roadrunner, just standing there, sticking his tongue out. I start to go after him, but I can only run in slow motion, about 1 meter per second. After one second, *the road stretches* ***uniformly*** *and instantaneously by 1 kilometer* so now that pesky fowl is 1998 meters away, since some of the stretch happens behind me. I try to speed up, but I'm still moving in slow motion, at 1 meter per second. After another second, the road stretches again by 1 kilometer so that now I'm 2995.5 meters away! And this just keeps on happening. Over, and over. And over. And over. Well, you get the idea. Then I wake up, hungry and frustrated.

I've gotta know: Do I ever get to the Roadrunner? Do I have any chance? If I do get there, how long does it take? Should I take a snack to eat along the way?

Not all of the dreams have these specific numbers. Sometimes the road is 2 km long and I'm moving at 3 meters per second; sometimes the road is 5 km long and I'm moving at 3 *centi*meters per second; and sometimes I don't know the exact length of the road or my precise speed. What I do know for certain in all of the dreams is that I'm moving at a constant slow rate and that the road stretches uniformly and instantaneously by its original amount after each second. You gotta help me figure out whether or not I get the silly bird in all these caes, and if so, how long it will take.

I know that your semester is winding down and you may be starting to get spring fever, but you've gotta give me an answer by April 16. I can't take this much longer.

Hungry as ever,
Wile E. Coyote

A Few Comments From Your Enterprising and Resourceful Professor

After reading Wile E. Coyote's sad tale, I have a couple of suggestions to help you get started.

- First, make sure you understand why the Roadrunner is 1998 meters away after the first stretch and 2995.5 meters away after the second stretch.

  Hint: The ***uniformity*** *of the stretch matters.*

- Next, set up a sequence $\{d_n\}$ where d_n represents the distance between Wile E. and the Roadrunner *after* n seconds, but *before* the road does its instantaneous stretch. i.e. immediately after the step but before the stretch.

  For example, $d_0 = 1000$, $d_1 = 999$, $d_2 = 1997$, $d_3 = 2994.5$, etc. (*Why?*)

- Then write

$$d_1 = 1 \cdot (\text{some expression involving } d_0)$$
$$d_2 = 2 \cdot (\text{some expression involving } d_1)$$
$$d_3 = 3 \cdot (\text{some expression involving } d_2)$$

  Now convert your expressions for d_2 and d_3 so that they only involve d_0.

- Use this to find a general expression for d_n in terms of d_0.

- Don't forget about the general case!