

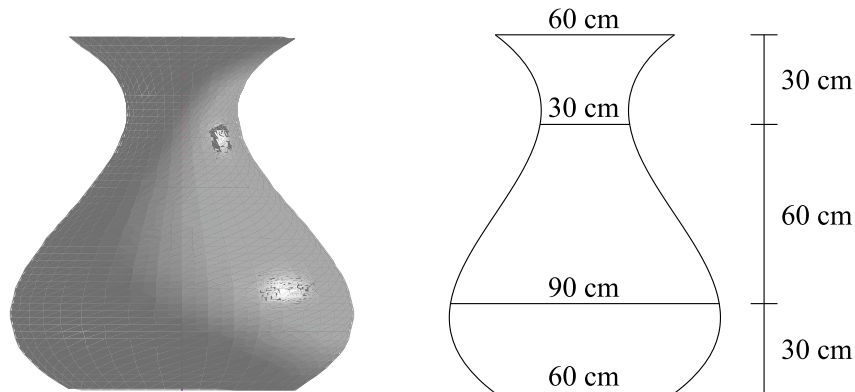
Joe Quimby
Springfield
February 24, 2010

Math 104 Students
Wheaton College
Norton, MA 02766

Dear Calculus Students:

These are tough, tough economic times, and even Springfield is affected by the ongoing crisis. With my reelection campaign about to kick off, I need to remind the good people of Springfield that I can be very successful in bringing federal funds to the town (although I can play the other side of screaming against the wasteful federal budget). I am confident that some of the not yet allocated federal stimulus funds will provide just what is needed. While some public officials may focus on the mundane (over-crowded schools, understaffed police and fire departments, crumbling bridges, unsafe water supply, aging power grid, blah-blah-blah), I think in much grander terms. Babylon may have had its Hanging Gardens, but I have commissioned Christo and Jeanne-Claude to design The Mayor Joe Quimby Floating Gardens of Springfield™. I'm hoping for a display that lasts longer than the two weeks that The Gates were up in Central Park, but those negotiations are a little tricky. The plans are mostly set, but there are still a few loose ends that need to be tied up before the project can be classified as "shovel ready" and eligible for funds under the terms of the stimulus. When I went looking for help, your enterprising and resourceful professor naturally referred me to you.

The basic idea is that we will have hundreds of large vases floating in Lake Springfield, each containing a tasteful botanical arrangement. As you may know, Lake Springfield is not necessarily the cleanest body of water in the United States, so the vases will be constructed of lightweight, one centimeter thick stainless steel. Also, the botanical arrangements will consist entirely of silk flowers and plastic plants to avoid the embarrassing wilting that occurs when any live vegetation gets too near Lake Springfield. The basic design of the vases is:



To meet the artistic vision of Christo and Jeanne-Claude, we will add play sand to the vases so that they will float at different heights, where 70 cm, 50 cm, or 20 cm of the vase is visible above the waterline. I have been promised that the vases will float, but after the whole experience with the Green Day concert, I don't want to take any chances. This is where I need your help. I need you to verify that the vases will float when empty, and I need to know how much sand to add to achieve each of the desired heights.

After describing my needs to your enterprising and resourceful professor, he suggested that you might find it useful to know that the stainless steel we will be using weighs approximately 4 grams per cubic centimeter, that play sand weighs approximately 1.5 grams per cubic centimeter, and that the weight of the silk arrangements is negligible.

In order to get at the front of the queue for the stimulus funds, I will need your report by the end of the day on March 5.

Mayorally yours,
Joe Quimby

A Few Comments From Your Enterprising and Resourceful Professor

- The vase looks suspiciously like a surface of revolution. You will probably want to find a function to model it, and since you have four points, a cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ might be a good place to start.
- It may also be useful to know that if an object floats, the object displaces an amount of water that is equal in weight to the weight of the object. i.e. A floating object that weighs 10 kilograms will displace 10 kilograms of water.
- You might find it interesting that one cubic centimeter of water weighs one gram.