Do the following series converge absolutely, converge conditionally, or diverge?

1.
$$\sum_{n=3}^{\infty} (-1)^{n+1} \frac{n^5}{n^6 - 5}$$

2.
$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k^2 + 1}$$

3.
$$\sum_{j=5}^{\infty} \frac{\sin(j) \ j!}{(j+2)!}$$

4.
$$\sum_{k=1}^{\infty} \frac{k}{e^k}$$