

While Visions of Roadrunners Danced in His Head

Wile E. Coyote
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Dear Calculus Students:

HELP ME!! For the last two months, I've had this series of recurring nightmares that are about to drive me out of my mind! When I went looking for help, your enterprising and resourceful professor naturally referred me to you.

The scenario is nearly always the same. I'm standing at the end of a road that is 1 kilometer long (for some reason the road has those little green kilometer markers on it), and there at the other end is that @!*~@#! Roadrunner, just standing there, sticking his tongue out. I start to go after him, but I can only run in slow motion, about 1 meter per second. After one second, *the road stretches uniformly and instantaneously by 1 kilometer* so now that pesky fowl is 1998 meters away, since some of the stretch happens behind me. I try to speed up, but I'm still moving in slow motion, at 1 meter per second. After another second, the road stretches again by 1 kilometer so that now I'm 2995.5 meters away! And this just keeps on happening. Over, and over. And over. And over. Well, you get the idea. Then I wake up, hungry and frustrated.

I've gotta know: Do I ever get to the Roadrunner? Do I have any chance? If I do get there, how long does it take? Should I take a snack to eat along the way?

Most of the dreams aren't that specific. Usually, I don't know how long the road is to begin with, or how fast I'm moving. All I know is that I'm always moving at the same slow rate, and the road stretches uniformly and instantaneously by its original amount after each second. You gotta help me figure out whether or not I get the silly bird, and if so, how long it will take.

I know that your semester is winding down and you may be starting to get spring fever, but you've gotta give me an answer by April 17. I can't take this much longer.

Hungry as ever,
Wile E. Coyote

A Few Comments From Your Enterprising and Resourceful Professor

After reading Wile E. Coyote's sad tale, I have a couple of suggestions to help you get started.

- First, make sure you understand why the Roadrunner is 1998 meters away after the first stretch and 2995.5 meters away after the second stretch.

*Hint: The **uniformity** of the stretch matters.*

- Next, set up a sequence $\{d_n\}$ where d_n represents the distance between Wile E. and the Roadrunner *after* n seconds, but *before* the road does its instantaneous stretch. i.e. immediately after the step but before the stretch.

For example, $d_0 = 1000$, $d_1 = 999$, $d_2 = 1997$, $d_3 = 2994.5$, etc. (*Why?*)

- Then write

$$d_1 = 1 \cdot (\text{some expression involving } d_0)$$

$$d_2 = 2 \cdot (\text{some expression involving } d_1)$$

$$d_3 = 3 \cdot (\text{some expression involving } d_2)$$

Now convert your expressions for d_2 and d_3 so that they only involve d_0 .

- Use this to find a general expression for d_n in terms of d_0 .
- Don't forget about the general case!