

1. Let $I = \int_0^1 x^4 \sin(x^2) dx$

- (a) Use a left sum to approximate I accurate within 0.001 of its actual value.
- (b) Use an infinite series to approximate I accurate within 0.001 of its actual value.

2. Find the volume of the solid formed when the graph of

$$f(x) = \frac{3x}{\sqrt{1+9x^6}} \text{ for } 0 \leq x \leq \frac{\pi}{4} \text{ is rotated about the } x\text{-axis.}$$

3. Do the following converge or diverge?

(a) $\int_2^{\infty} x^3 e^{-x^2} dx$

(c) $\sum_{k=1}^{\infty} (-1)^k \frac{3k^2}{8k^3 - 8}$

(b) $\int_1^{\infty} \frac{3^x}{4^x + 7} dx$

(d) $\sum_{j=1}^{\infty} \frac{3j^2}{j!}$

4. Show that $\int_4^{\infty} \frac{3x^3}{2x^5 + \ln(x)} dx$ converges and find a definite integral I_1 that approximates I accurate within 0.01.
5. Show $\sum_{k=4}^{\infty} \frac{3 \cos(k)}{2k^2 + \sin(k)^2}$ converges absolutely and find an N such that S_N approximates I accurate within 0.01.
6. Let $I = \int_0^4 \sqrt{1 + (xe^{-x^2})^2} dx$.

Explain how I can be interpreted as:

- ▶ a one-dimensional measure corresponding to a function $f(x)$,
- ▶ a two-dimensional measure corresponding to a function $g(x)$,
- ▶ and as a three-dimensional measure corresponding to a function $h(x)$.

Be sure to give formulas for $f(x)$, $g(x)$ and $h(x)$.