

Let $f(x, y) = (P(x, y), Q(x, y)) = (y, -x)$.

Let γ be the unit circle oriented counterclockwise and let R be the region enclosed by γ .

Calculate the following

$$1. \int_{\gamma} P \, dx + Q \, dy = \int_{\gamma} f(X) \cdot dX$$

$$2. \int \int_R Q_x - P_y \, dA$$

Use Green's Theorem to evaluate $\int_{\gamma} f(X) \cdot dX$ in the following cases:

1. $f(x, y) = (y^2 + x^2, x + y)$; γ is the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$
2. $f(x, y) = (y^2 + x, x + y)$; γ is the circle of radius 1 centered at the origin
3. $f(x, y) = (x - y, x + y)$; γ is the circle of radius 3 with center $(1, 0)$