1. Let $I = \int_0^3 e^{-x^4}\,dx$.

   (a) Check that Theorem 1 applies, and use this to find an n so that R_n approximates I within 10^{-6} of its actual value.

   (b) Now use Theorem 2 to find an n so that R_n approximates I within 10^{-6} of its actual value.

   (c) Use Theorem 3 to find an n so that M_n approximates I within 10^{-6} of its actual value. Calculate M_n for this value of n.

2. Let $I = \int_0^2 \sqrt{4 - x^2}\,dx$.

   (a) Check that Theorem 1 applies, and use this to find an n so that L_n approximates I within 0.001 of its actual value.

   (b) Now try to use use Theorem 2 to find an n so that L_n approximates I within 0.001 of its actual value. (Look very closely near $x = 2$)
   What's happening? Why?

   (c) What is the *exact* value of I?

Recap for Today

Usually Theorems 2 and 3 give you better error bounds than Theorem 1, but not always.