Some Useful Maple Commands for
A Lava Lamp for the New Millenium™

Piecewise Defined Functions

You will probably want to use different functions over different intervals to design your lava lamp. To define the function

$$f(x) = \begin{cases} x+3 & 0 \le x < 1 \\ 5-x & 1 \le x \le 3 \end{cases}$$

in Maple, use the following command

```
f := x -> piecewise( 0<=x and x<1, x+3, 1<x and x<=3, 5-x);
```

Then you can plot $f(x)$ and rotate it about the x-axis:

```
plot(f(x), x=0..3);
with(plots);
tubeplot( [x,0,0], x=0..3, radius= f(x));
```

Smoothing the Corners

Notice that there is a sharp edge on this surface, which isn't allowed. To smooth this out, we'll cut out the sharp corner and find a cubic function that matches the pieces smoothly.

We'll change $f(x)$ to be

$$f(x) = \begin{cases} x+3 & 0 \le x < 0.5 \\ g(x) & x \le 0.5 < 1.5 \\ 5-x & 1.5 \le x \le 3 \end{cases}$$

where $g(x) = ax^3 + bx^2 + cx + d$ where $g(x)$ matches $f(x)$ at both $x = 0.5$ and $x = 1.5$ *and* $g(x)$ has the same derivative as $f(x)$ at these two points. That is, we want to find values for a, b, c, and d so that

$$g(0.5) = 3.5, \qquad g'(0.5) = 1, \qquad g(1.5) = 3.5, \qquad g'(1.5) = -1$$

The simplest way to do this in Maple is the following.

```
g := x -> a*x^3 + b*x^2 + c*x + d;
dg := x -> 3*a*x^2 + 2*b*x + c;
solve( { g(0.5)=3.5, dg(0.5)=1, g(1.5)=3.5, dg(1.5)=-1} );
```

This should give you values $a = 0, b = -1, c = 2, d = 2.75$. Therefore, our smoothed function is

$$f(x) = \begin{cases} x+3 & 0 \le x < 0.5 \\ -x^2 + 2x + 2.75 & x \le 0.5 < 1.5 \\ 5-x & 1.5 \le x \le 3 \end{cases}$$

Try graphing this to verify that the corners are smooth now.

Notice that you don't have to re-enter the definitions of g and dg from now on when you want to smooth other corners.