PROBLEM SET #3

Due Thursday, September 17, 2020 @ midnight
Submit as single pdf file to onCourse

1. Use the Intermediate Value Theorem to show that $f(x) = 3\cos(x^2 - 1) + x^2 - x$ has a root between $x = 1$ and $x = 2$. Be sure to explain your answer.

2. Let $g(x) = \dfrac{2x^2 - 6x - 8}{5x^2 - 25x + 20}$

   (a) Find $\lim\limits_{x \to 1^+} g(x)$

   (b) Find $\lim\limits_{x \to 4} g(x)$

   (c) Find $\lim\limits_{x \to \infty} g(x)$

3. Use function $g(x)$ from #2 and your answers to #2 to answer the following.

   (a) Does $g(x)$ have any vertical asymptotes? If so, where?

   (b) Does $g(x)$ have any horizontal asymptotes? If so, where?

   (c) Is $g(x)$ continuous at $x = 4$? Explain.

4. Let $f(x) = x^3 + x + 1$

   (a) Use the limit definition of the derivative to find $f'(1)$

   (b) Use your answer to part (a) to find the equation of the line tangent to the graph of $y = f(x)$ at $x = 1$

   (c) Verify your answer by graphing $y = f(x)$ and your line from part (b) on the same set of axes. Include your graph with your solutions.