

$$1. \text{ If } A = \begin{bmatrix} 1 & 24 & -13 & -12 \\ 1 & 3 & -2 & -1 \\ 7 & 0 & -3 & 4 \end{bmatrix}, \text{ then } \text{REF}(A) = \begin{bmatrix} 1 & 0 & -\frac{3}{7} & \frac{4}{7} \\ 0 & 1 & -\frac{11}{21} & -\frac{11}{21} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Find bases for $\text{col}(A)$, $\text{nul}(A)$, and $\text{row}(A)$.

- If A is 5×9 of rank 4, what is the dimension of $\text{nul}(A)$?
- If A is the matrix corresponding to a one-one linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^7$, what is the dimension of $\text{nul}(A)$? of $\text{row}(A)$? of $\text{nul}(A^T)$?
- Suppose that A is $m \times n$ where $A\mathbf{x} = \mathbf{b}$ is consistent for all $\mathbf{b} \in \mathbb{R}^m$. How many solutions does $A^T\mathbf{y} = 0$ have?