

Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$.

1. Verify that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthogonal basis for \mathbb{R}^3 .
2. Find $\hat{\mathbf{y}}_1$, the orthogonal projection of \mathbf{y} onto \mathbf{u}_1 .
3. Find $\hat{\mathbf{y}}_2$, the orthogonal projection of \mathbf{y} onto \mathbf{u}_2 .
4. Find $\hat{\mathbf{y}}_3$, the orthogonal projection of \mathbf{y} onto \mathbf{u}_3 .
5. Verify that $\mathbf{y} = \hat{\mathbf{y}}_1 + \hat{\mathbf{y}}_2 + \hat{\mathbf{y}}_3$.