

Let  $g(x, y) = x^2 - 4x + y^2 - 8y + xy + 20$ .

1. Find  $g_x$  and  $g_y$ .
2. Evaluate  $g_x(1, 3)$  and  $g_y(1, 3)$ .
3. Create a 3-D plot of  $z = g(x, y)$  near  $(1, 3)$ .  
Do your answers from #2 make sense?
4. At what point  $(x_0, y_0)$  does  $z = g(x, y)$  obtain its minimum value?  
Verify your answer using a contour plot of  $g(x, y)$ .

1. What is the largest volume for a box that is to be constructed from 96 square inches of cardboard?  
What are the dimensions of the box that give this largest volume?
2. How does your answer from #1 change if it is an open box without a top?
3. How does your answer from #1 change if the bottom of the box needs to be reinforced with a double layer of cardboard?