

Theorem 6 (1984, pg 72, Saari)

Suppose there are $n \geq 3$ candidates.

- Rank the n candidates however you like.
Pick a positional method for the n candidate election.
- Drop 1 candidate, rank the remaining $n - 1$ however you like.
Pick a method for these $n - 1$ candidates.
- Continue dropping and picking until have 2 candidates.
Use majority rule to select winner for the final 2.

There exists a profile so that when voters vote on these sets using these methods, you get the indicated outcome.

Theorem 7 (1989, pg 77, Saari)

Suppose there are $N \geq 3$ candidates

- Rank the candidates in any desired transitive manner and select a positional method.
- For each of the N ways to drop one candidate, pick a transitive ranking for the remaining $N - 1$ candidates.
For each subset of $N - 1$ candidates, pick a positional method.
- For each subset of $N - 1$ candidates, there are $N - 1$ ways to drop a candidate leaving a subset of $N - 2$ candidates.
Pick an ordering for each of these and a positional method.
- Continue until you are left with pairs of candidates.
Use the majority vote for the pairs.

For almost all choices of positional voting methods (including plurality, vote for two, vote for three, etc.) there exists a profile so that when the ballots are tallied on the subsets using the indicated method, the outcome corresponds to the specified ranking.

The Borda Count is a method that does not allow this type of outcome.

Calculate the winner under

- Plurality
- Vote for two
- Vote for three
- Antiplurality (vote for four since five candidates)
- The Borda Count

Number	Preference
3	$A > B > C > D > E$
1	$A > C > E > D > B$
2	$A > E > C > D > B$
2	$C > B > D > E > A$
2	$D > C > E > A > B$
1	$E > A > C > D > B$
3	$E > B > D > A > C$

Theorem 2 (1992, pg 36, Saari)

For $N \geq 3$ candidates $\{c_1, c_2, \dots, c_N\}$, there exist profiles such that c_j wins when the voters vote for j candidates $j = 1, \dots, N - 1$ and then c_N is the Borda Count winner.

Theorem 3 (1992, pg 37, Saari)

Suppose there are $N \geq 2$ candidates. For any k such that

$$1 \leq k \leq N! - (N - 1)!$$

there exists a profile where there are exactly k strict positional election outcomes.

The different rankings come by changing the positional method used.

It is impossible to find a profile with more than $N! - (N - 1)!$ strict positional rankings.