

Homework #4, Due November 1, 2007
Individual Assignment

1. (20 pts) Create examples of profiles with three candidates that have the following properties, or explain why no such profile exists. If the profile exists, for each outcome, give the weights for a procedure that determines that outcome.

(a) All positional methods give the ranking of $A \succ B \succ C$.

(b) By varying the positional procedure, the profile gives **only** the following *strict* positional outcomes (Note that other outcomes involving ties are allowed) :

$$A \succ B \succ C, \quad A \succ C \succ B, \quad C \succ A \succ B$$

(c) Repeat part (b) but with outcomes

$$C \succ B \succ A, \quad B \succ C \succ A, \quad B \succ A \succ C, \quad A \succ B \succ C$$

(d) Repeat part (b) but with outcomes

$$C \succ A \succ B, \quad C \succ B \succ A, \quad A \succ B \succ C$$

2. (20 pts) Let $q_0 = (a_0, b_0, c_0)$ denote the plurality point in the representation triangle for a profile and $q_1 = (a_1, b_1, c_1)$ denote the antiplurality point.

(a) Show that

$$\frac{1}{2}a_0 \leq a_1 \leq \frac{1}{2}, \quad \frac{1}{2}b_0 \leq b_1 \leq \frac{1}{2}, \quad \text{and} \quad \frac{1}{2}c_0 \leq c_1 \leq \frac{1}{2}$$

(b) In each case, create a profile with these outcomes or explain why it is impossible.

i. $q_0 = (\frac{1}{32}, \frac{2}{3}, \frac{29}{96})$ $q_1 = (\frac{1}{5}, \frac{8}{15}, \frac{4}{15})$

ii. $q_0 = (\frac{1}{4}, \frac{5}{12}, \frac{1}{3})$ $q_1 = (\frac{3}{8}, \frac{1}{4}, \frac{3}{8})$

iii. $q_0 = (\frac{3}{22}, \frac{5}{22}, \frac{7}{11})$ $q_1 = (\frac{3}{7}, \frac{3}{7}, \frac{1}{7})$

(c) For each profile that you determined was possible in (b), give all possible outcomes for this profile (including ties) using a positional method.