

Why the product rule is true

$$= \frac{d}{dx} (f(x) g(x))$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) g(x+h) - f(x) g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) g(x+h) - \textcolor{red}{f(x)} g(x+h) + \textcolor{red}{f(x)} g(x+h) - f(x) g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \left(\frac{f(x+h) \textcolor{blue}{g(x+h)} - f(x) \textcolor{blue}{g(x+h)}}{h} + \frac{\textcolor{blue}{f(x)} g(x+h) - \textcolor{blue}{f(x)} g(x)}{h} \right)$$

$$= \lim_{h \rightarrow 0} \left(\left[\frac{f(x+h) - f(x)}{h} \right] \textcolor{blue}{g(x+h)} + \textcolor{blue}{f(x)} \left[\frac{g(x+h) - g(x)}{h} \right] \right)$$

$$= f'(x) \textcolor{blue}{g(x)} + \textcolor{blue}{f(x)} g'(x)$$

For each function, find its derivative.

1. $f(x) = e^x \sin(x)$

2. $f(x) = \ln(x)(x^3 - 3)$

3. $f(x) = \frac{x + \cos(x)}{x^2}$

4. $f(x) = \tan(x)$ Hint: $\tan(x) = \frac{\sin(x)}{\cos(x)}$

5. $f(x) = \frac{3 \ln(x)}{e^x}$