Why the product rule is true

$$= \frac{d}{dx}\left(f(x)\, g(x)\right)$$

$$= \lim_{h\to 0} \frac{f(x+h)\, g(x+h) - f(x)\, g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\, g(x+h) - f(x)\, g(x+h) + f(x)\, g(x+h) - f(x)\, g(x)}{h}$$

$$= \lim_{h\to 0} \left(\frac{f(x+h)\, g(x+h) - f(x)\, g(x+h)}{h} + \frac{f(x)\, g(x+h) - f(x)\, g(x)}{h} \right)$$

$$= \lim_{h\to 0} \left(\left[\frac{f(x+h) - f(x)}{h} \right] g(x+h) \;+\; f(x) \left[\frac{g(x+h) - g(x)}{h} \right] \right)$$

$$= f'(x)g(x) + f(x)g'(x)$$

For each function, find its derivative.

1. $f(x) = e^x \sin(x)$

2. $f(x) = \ln(x)(x^3 - 3)$

3. $f(x) = \dfrac{x + \cos(x)}{x^2}$

4. $f(x) = \tan(x)$ Hint: $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$

5. $f(x) = \dfrac{3\ln(x)}{e^x}$